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# Introduction to ''Numerical Solution of the Quasilinear Poisson Equation in a Nonuniform Triangle Mesh''

Alan Winslow's article has had enormous influence on the development of grid generation techniques. It contains ideas that are fundamental to the entire field of elliptic grid generation. However, the paper is seldom cited because it mostly deals with how to write difference equations on a triangular grid, and the key ideas on grid generation are hidden in a four-page appendix. Most citations refer to the more modern work of Thompson, Warsi, and Mastin [1], which popularized body-fitted, quadrilateral grids.

To those who have derived or used body-fitted structured grids, it is clear that Winslow was the first to cover the key ideas for elliptic grid generation. On the requirement for a logical reference grid ''The type of triangulation used here is topologically regular; that is, it is topologically equivalent to an equilateral triangle array in which six triangles meet at every interior mesh point'' (red quadrilateral, and the statement is just as true). On the form of the grid generator equations: ''A satisfactory method can be derived by formulating the zoning problem as a potential problem, with mesh lines playing the role of equipotentials.'' On the properties of the grid: ''Because of the well-known averaging property of solutions of Laplace's equation, we might expect a mesh constructed in this way to be, in some sense, smooth.'' On the choice of dependent variables: ''The generator equations can be solved numerically by inverting them and writing them in terms of $x(\chi, \psi)$ and $y(\chi, \psi)$.'' All of these ideas were essential to the success of body-fitted grids [1].

Evidently, the ideas contained in Winslow's paper had a long gestation. In an unpublished report in 1963 [2], Winslow said, ''It was pointed out several years ago by N. Hardy and C. E. Leith (unpublished) that harmonic functions, because of their well-known averaging property, would be particularly suitable for the construction of smooth meshes.'' ''In 1962, W. P. Crowley (unpublished) developed the idea further by defining two functions $\phi(x, y)$ and $\psi(x, y)$, each of which satisfied Laplace's equation.'' Winslow's report presents Crowley's method in the simpler form that later became well known through Thompson's work [1, 3]. Alan Winslow's *Journal of Computational Physics* article was the first appearance of these equations in published form.

Winslow's approach is not unique. The contemporary paper of Godunov and Propokov [4], for example, considered an equidistribution functional which yields quite different Euler equations. It is Winslow's approach, however, that has stood the test of time.

Many key advances in structured grid generation have been reported by others in the years since. Some of the more prominent ones include the transformation of the Navier–Stokes equations by Gal-Chen and Somerville [5], iterative adjustment of control functions for boundary orthogonality by Steger and Sorenson [6], variational methods for adaptive grids by Brackbill and Saltzman [7], orthogonal grids by Ryskin and Leal [8], block-structured grids by Miki and Takagi [9], surface grid equations by Warsi [10], overset grids by Chesshire and Henshaw [11], and the most recent advance in control functions by Spekreijse [12]. There are, of course, other approaches to grid generation, among them the very successful adaptive mesh refinement algorithm [13].

By now, grid generation has become an essential part of computational fluid dynamics, electromagnetics, and the like. Its use gives to finite difference and finite volume methods capabilities for modeling realistic geometries similar to those of the finite element method and extends computational modeling of fluid flow to engineering analysis and design. This ability to treat general real-world geometric configurations has opened the door for finite difference and finite volume computational fluid modeling to become a major part of the ''computational paradigm''—joining the experimental and theoretical modes to elucidate physical phenomena and processes: to significantly shorten the design cycle and to usher in multidisciplinary design optimization.

Of course, the possiblities of elliptic structured grid generation are by now well explored. New methods with new capabilities have been developed. Nevertheless, Winslow deserves great credit for pioneering the field and for providing us the first practical way to use computational grids more effectively.

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